Deflection Of Multilayered Laminated Antisymmetric Composite Plate By Using Different Structural Theories

Zeshan Ahmad[1], Dr. Sabah Khan[2], Hasan Zakir Jafri[3], Furkan Ahmad[4]
response.zeeshan@gmail.com[1], Khan_sabah@yahoo.com[2], hasan.jafri@rediffmail.com[3], furkanme13@gmail.com[4]
[1,3]Research Scholar, Jamia Millia Islamia, New Delhi
[2]Associate Professor, Jamia Millia Islamia, New Delhi
[4] M.Tech Student, Netaji Subhas Institute of Technology, New Delhi

Abstract: *Abstract:* In this paper, we adopt classical,first-order shear deformation and higher order deformation theories to calculate *non dimensional deflection. Two type of loading such as uniformly distributed and static sinusoidal distributed loads are used. The non dimensional deflection are calculated at mid plane and at various nodes by using different structural theories. Glass-epoxy laminated composite material used for this analysis, because it has good structural properties.In these theories, the material properties of the constituent layers are combined to form a hypothetical single layer whose properties are equivalent to-through-thickness integrated sum of its constituents. The results of non-dimensional deflection obtained by different theories are compared for different aspect ratio with the variation of side to thickness ratio.*

Keywords: Laminate composite plate, Non- dimensional deflection, structural theories

.

1 Introduction

Composite laminates are being increasingly used in a number of areas including aerospace, automobile, nuclear, marine, civil and biomedical engineering. This is mainly due to their high stiffness and strength-to-weight ratios and other superior material characteristic such as long fatigue life and ability to manipulate fibre orientations to meet design requirements. However, composite laminates are at the same time significantly weak in shear, which demand special attention on the part of designers. Chai et al. studied one dimensional model for the analysis of delamination buckling of beam-plate in 1981 [1]. Shivakumar et al. investigated the buckling behavior of thin elliptical delamination using the Rayleigh–Ritz and finite element method [2]. F Auricchio et al presented 4-node finite element for the analysis of laminated composite plates with monoclinic layers[3]. D Samaratunga et al. presented Wave propagation analysis in laminated composite plates with transverse cracks using the wavelet spectral finite element method [4]. A Szekrenyes proposed semi layer wise approach captures the mechanical behavior of delaminated composite plates using four equivalent single layers independently of the lay-up[5]. P Dey et al developed a model for the analysis of laminated composite plates. The element was capable of representing high orders of displacement using very few degrees of freedom. Performance of the element was tested in a wide range of problems which indicate that combining all these features can help to achieve great accuracy at reduced computation cost [6]. The aim of this paper is to calculate the non-dimensional deflections at mid plane and at various nodes for antisymmetric plate. The non-dimensional are calculated for different loading conditions (viz uniformly distributed and static sinusoidal distributed load). Finally the model is implemented by Finite Element Method on a laminated composite plate of any lay-up.

2 Methodology

The equivalent single-layer laminate theories are those in which a heterogeneous laminated plate is treated as a statically equivalent, single layer having a complex constitutive behaviour, reducing the 3-D continuum problem to a 2-D problem. The ESL theories are developed by assuming the form of the displacement field or stress field as a linear combination of unknown functions and the thickness coordinate:
e heat conduction problem in the casting and transient surface

$$\varphi_i(x,y,z,t)=\sum_{j=0}^{N}(z)^j\,\varphi_i^j\,(x,y,z,t)$$

Where
φ_i=ith component of displacement or stress,
(x, y)=in plane coordinates
z=the thickness coordinate
t=time,
and φ_i are determined by the principle of virtual displacement (or its dynamics version when time dependency is be included):

$$\int_0^T(\delta U+\delta V-\delta K)dt=0$$

Where

δU = Virtual strain energy,
δU = Virtual work done by external applied forces,
δK = Virtual kinetics
The simplest ESM laminate theory is the classical laminate plate theory (or CLPT), which is an extension of the Kirchhoff

(classical) plate theory to lamina composite plates. It is based on the displacement field.

2.1 The Classical Laminated Plate Theory:

Assumptions:

(1) Straight lines perpendicular to the mid surfaces (i.e., transverse normal) before deformation remain straight after deformation.

(2) The transverse normal do not experienced elongation (i.e., they are inextensible).

(3) The transverse normal rotates such that they remain perpendicular to the mid surfaces after deformation.

This theory is based on the displacement field. It is given as follow:

$$u(x,y,z,t) = u_0(x,y,t) - z\frac{\partial w_0}{\partial x}$$

$$v(x,y,z,t) = v_0(x,y,t) - z\frac{\partial w_0}{\partial y}$$

$$w(x,y,z,t) = w_0(x,y,t)$$

Where $(u_{0,}v_{0,}w_0)$ are the displacement components along the (x, y, z) coordinate directions respectively, of a point on the mid plane (i.e., z = 0).

2.2 First Order Shear Deformation theory (FSDT):

The next theory in the hierarchy of ESL laminates theories is the first order shear deformation theory (or FSDT), which is also based on the displacement field:

$$u(x,y,z,t) = u_0(x,y,t) + z\emptyset_x(x,y,t)$$

$$v(x,y,z,t) = v_0(x,y,t) + z\emptyset_y(x,y,t)$$

$$w(x,y,z,t) = w_0(x,y,t)$$

Where $\emptyset_x$ and $\emptyset_y$ denote rotations about the y & x axes respectively.

2.3 Third-Order Shear Deformation Theory (TSDT):

The third-order plate theory to be developed is based on the same assumptions as the classical and first-order plate theories, except that we relax the assumption on the straightness and normality of a transverse normal effect deformation by expanding displacement (u,v,w) as cubic functions of the thickness coordinate. The thickness of deformations of the transverse normal of edge y =0.

The third-order laminate with transverse inextensibility is based on the displacement field

$$u(x,y,z,t) = u_0(x,y,t) + z\emptyset_x(x,y,t) + z^2\Psi_x(x,y,t) + z^3\left(-\frac{4}{3h^2}\right)\left(\emptyset_x + \frac{\partial w_0}{\partial x}\right)$$

$$v(x,y,z,t) = v_0(x,y,t) + z\emptyset_y(x,y,t) + z^2\Psi_y(x,y,t) + z^3\left(-\frac{4}{3h^2}\right)\left(\emptyset_y + \frac{\partial w_0}{\partial y}\right)$$

$$w(x,y,z,t) = w_0(x,y,t)$$

The displacement field accommodates quadratic variation of transverse shear strain (and hence stresses) and vanishing of transverse shear stresses on the top and bottom of a general laminate composed of monoclinic layers. Thus there is no need to use shear correction factors in a third order theory.

2.4 Layerwise Theory:

In contrast to the ESL theories, the layerwise theories are developed by assuming that the displacement field exhibits only C^0-continuity through the laminate thickness. Thus the displacement components are continuous through the laminate thickness but the derivatives of the displacements with respect to the thickness coordinate may be discontinuous at various points through the thickness, thereby allowing for the possibility of continuous transverse stresses at interfaces separating dissimilar materials. Layerwise displacement field provide a much more kinematically correct representation of the moderate to severe cross-sectional warping associated with the deformation of thick laminates.

The displacement-based layerwise theories can be subdivided into two classes:

(1). The partial layerwise theories that use layerwise expansions for the in–plane displacement components but not the transverse displacement component.

(2). Full layerwise theories that use layerwise expansions for all three displacement components

The total displacement field of the laminate can be written as

$$u(x,y,z) = \sum_{I=1}^{N} U^I(x,y)\cdot\varphi^I(z)$$

$$v(x,y,z) = \sum_{I=1}^{N} V^I(x,y) \cdot \varphi^I(z)$$

$$w(x,y,z) = \sum_{I=1}^{N} W^I(x,y) \cdot \psi^I(z)$$

Where (U^I, V^I, W^I) denote the nodal values of (u, v, w) , N_e is the number of nodes and φ^I are the global interpolation functions for the discretization of the in-plane displacements through thickness, and M is the number of nodes and ψ^I are the global interpolation functions for discretization of the transverse displacement through thickness. For linear and quadratic variation through each numerical layer these function are given below.

Where N = number of layers through the thickness

Linear functions $(N_e = N + 1)$

$$\varphi^1(z) = \psi_1^{(1)}(z) \qquad z_1 \le z \le z_2$$

$$\varphi^I(z) = \begin{cases} \psi_2^{(I-1)}(z) & z_{I-1} \le z \le z_I \\ \psi_2^{(I-1)}(z) & z_1 \le z \le z_{I+1} \end{cases} \quad (I = 2,3 \ldots\ldots N-1) \varphi^N(z) = \psi_1^{(N)}(z) \qquad z_{N-1} \le z \le z_N$$

$$\psi_1^{(k)} = 1 - \frac{\bar{z}}{h_k} \qquad \psi_2^{(k)} = \frac{\bar{z}}{h_k} \qquad 0 \le \bar{z} \le h_k$$

Quadratic functions $(N_e = 2N + 1)$

$$\varphi^1(z) = \psi_1^{(1)}(z) \qquad z_1 \le z \le z_3$$

$$\varphi^{2I}(z) = \psi_1^{(I)}(z) \qquad z_{2I-1} \le z \le z_{2I+1}$$

$$\varphi^{2I+1}(z) = \begin{cases} \psi_3^{(I)}(z) & z_{2I-1} \le z \le z_I \\ \psi_1^{(I+1)}(z) & z_{2I+1} \le z \le z_{2I+3} \end{cases} \quad (I = 2,3 \ldots\ldots N-1)$$

$$\varphi^N(z) = \psi_3^{(N)}(z) \qquad z_{N-1} \le z \le z_N$$

$$\psi_1^{(k)} = \left(1 - \frac{\bar{z}}{h_k}\right)\left(1 - \frac{2z}{h_k}\right) \qquad \psi_2^{(k)} = 4\frac{z}{h_k}\left(1 - \frac{z}{h_k}\right)$$

$$z_{2k-1} \le z \le z_{2k+1}$$

Where h_k is the thickness of the k-th layer, $\bar{z} = z - z_t^k$, z_t^k denotes the z-coordinate of the top of the k-th numerical. Independent approximations for the in-plane and transverse displacements are assumed in order to include the possibility of inextensibility of transverse normal. The inextensibility of transverse normal can be included by setting M =1 and ψ^I = 1for all z.

Stacking Sequence

A laminate is a collection of laminae stacked to achieve the desired stiffness and thickness. The sequence of various orientation of a fiber reinforced composite layer in a laminate is termed the lamination scheme or stacking sequence. In our problem we have assumed four and eight layers laminate with symmetric lamination scheme.

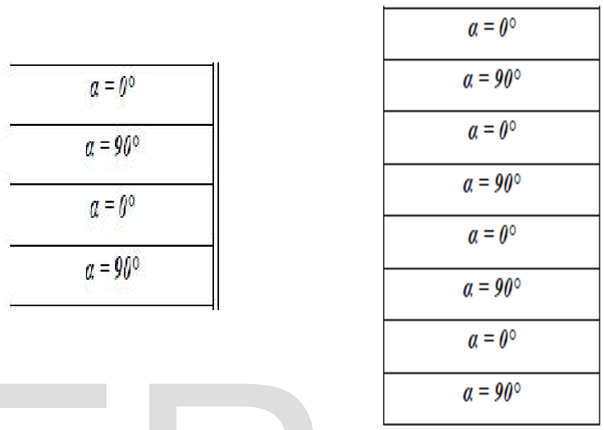

Fig.1: Stacking sequence

3 RESULTS AND DISCUSSION

The model developed herein was run for a laminate plate. The properties of assumed material of laminated plate are in table.

Table 1: Material property

Young's moduli (Gpa)			Shear moduli (Gpa)			Poisson ratio			Density Kg/m^3
E_1	E_2 (a). Geometry	E_3	G_{23}	G_{13}	G_{12}	ν_{12}	ν_{13}	ν_{23}	ρ
175	7	7	1.4	3.5	3.5	0.25	0.25	0.25	1600

The non-dimensional transverse deflection are given by

$$\bar{w} = w_0(x,y)\left(\frac{E_2 h^3}{a^4 q_0}\right)$$

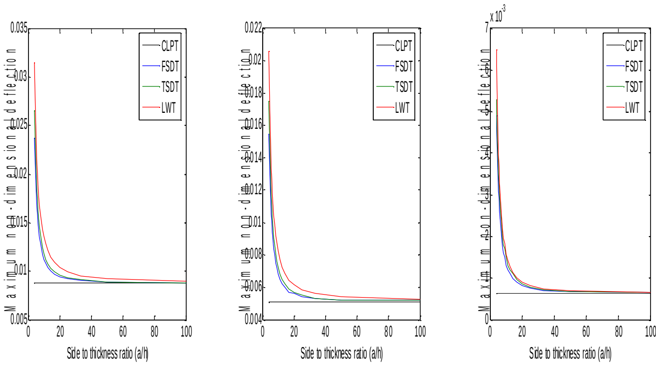

Fig. 2: Comparison of maximum non-dimensional deflection subjected to sinusoidal distributed load for the eight layers anti-symmetric laminates with different aspect ratio: (a). Less than uni(b). Equal than unity(c). Greater than unity

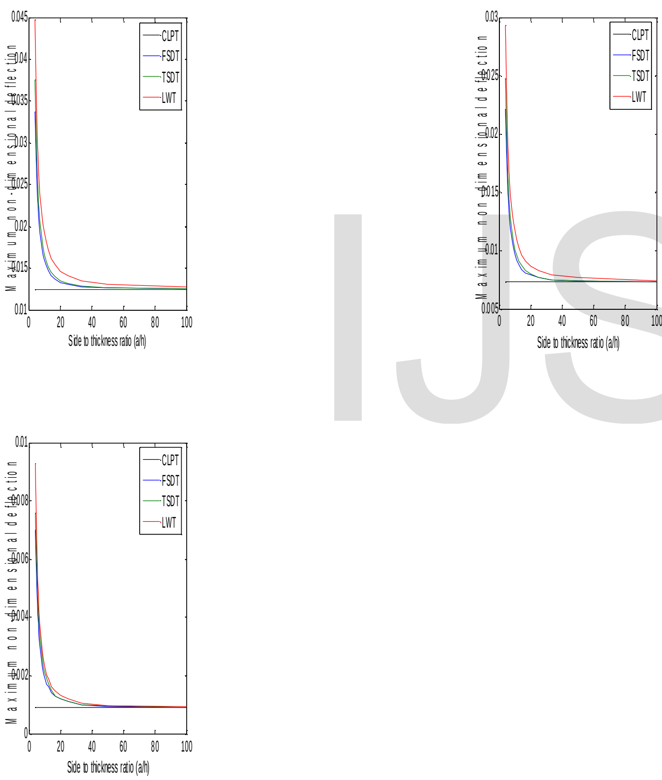

Fig 3: Comparison of maximum non-dimensional subjected to uniformly distributed load deflection for the eight layers symmetric laminates with different aspect ratio:

(a). Less than unity (b). Equal than unity (c). Greater than unity

Anti-symmetric cross-ply laminate subjected to sinusoidal distributed load

Figure 2 show the plots of maximum non-dimensional deflection ($\overline{w}$) versus side to thickness ratio for anti-symmetric cross-ply with orientation (0˚/90˚/0˚/90˚) laminates subjected to sinusoidal distributed load for different aspect ratio. In this case, the plots of maximum non-dimensional deflection are also curvilinear for the LWT, TSDT and FSDT theory, and the plot of CLPT is a straight line as in the case of symmetric cross-ply laminated plate

The same analysis has been carried out for the eight layers anti-symmetric cross-ply with orientation (0˚/90˚/0˚/ 90˚ /0˚/90˚/0˚/90˚) laminate subjected to sinusoidal distributed load. The plots of 2D non-dimensional deflection are also in same nature (as shown in fig. 3) as the four layers anti-symmetric cross-ply laminate but the results of non-dimensional deflection are less than the previous case.

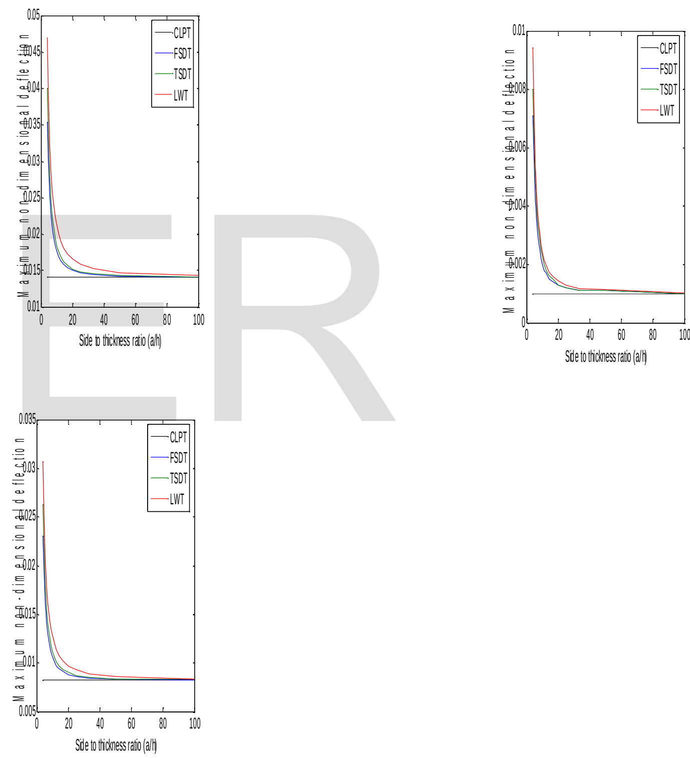

Fig: 4: Comparison of maximum non-dimensional subjected to uniformly distributed load deflection for the four layers anti-symmetric laminates with different aspect ratio:

(a). Less than unity (b). Equal than unity (c). Greater than unity

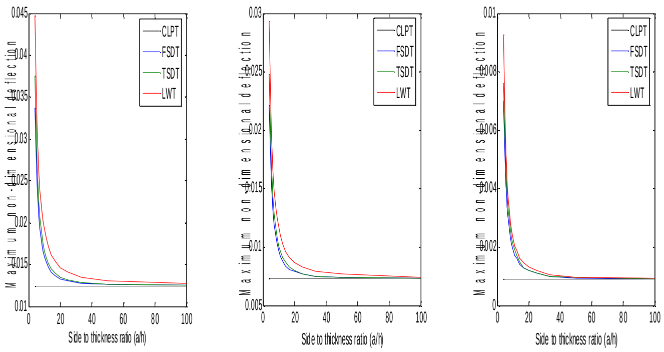

Fig 5: Comparison of maximum non-dimensional subjected to uniformly distributed load deflection for the eight layers symmetric laminates with different aspect ratio:

(a). Less than unity (b). Equal than unity (c). Greater than unity

Anti-symmetric cross-ply laminate subjected to uniformly distributed load

A rectangular cross-ply laminated plate subjected to uniformly distributed load is considered for different aspect ratio varying from 0.5 to 1.5 in this example. The stacking sequences and the boundary conditions are identical as taken in previous cases. The analysis is made for four layers and eight layers anti-symmetric cross-ply laminated plate taking side to thickness ratio from 4 to 100. Fig 4 and 5 show the 2D plots of maximum non-dimensional deflection versus side to thickness ratio by using given theories for different aspect ratio. The results obtained are in good agreement to each other. The non-dimensional deflection is greater for uniformly distributed load as compared to sinusoidal distributed load for both four layers and eight layers.

4 Conclusion

This study considers the non-dimensional deflection response of laminated composite rectangular plates with simply supported boundary conditions. From the present analytical study, it was noted that different side to thickness ratio affected the non-dimensional deflection. The non-dimensional deflection decreases as side to thickness ratio increases. As the aspect ratio increases, the non-dimensional deflection of the plate decreases. When the aspect ratio changed from less than unity to greater than unity, the variation in the non-dimensional deflection is very large. It was seen that the different stacking sequence affected the non-dimensional deflection. When the stacking sequence changes from symmetric to anti-symmetric, the non-dimensional deflection decreases. The non-dimensional deflection due to the uniformly distributed load is found to be greater than sinusoidal distributed load.

References

[1] Chai H, Babcock CD, Knauss WG, "One dimensional modeling of failure in laminated plates by delamination buckling. Int J Solids Struct, vol. 17, pp. 1069–83, 1981.

[2] Shivakumar KN, Withcomb JD, " Buckling of sublaminate in a quasi isotropic composite laminates" J Compos Mater; vol. 19, pp. :2–18, 1985.

[3] Ferdinando Auricchio, Elio Sacco, Giuseppe Vairo, "A mixed FSDT finite element for monoclinic laminated plates", Computers and Structures, Vol. 84 , pp.624–639, 2006.

[4] D. Samaratunga, R Jha, S. Gopalkrishan, " Wave propagation analysis in laminated composite plate with transverse cracks using the wavelets spectral finite element method", Finite element in analysis and design, vol. 89, pp. 19-32, 2014

[5] AndrásSzekrényes, "Semi layerwiseanalysisoflaminatedplateswithnonsingulardelamination –The theorem of autocontinuity", Applied Mathematical Modelling, vol. 1, pp. 1–28, 2015.

[6] P.Dey, A.H.Sheikh, D.Sengupta, "A new element for the analysis of composite plates" Finite ElementsinAnalysisandDesign VOL. 82, PP. 62–71, 2014.